Classical rotons due to light fluctuation and plasma-like effects in cold atomic traps

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In large magneto-optical traps (MOTs), up to $10^9 - 10^{10}$ atoms can be stored with the help of light and magnetic fields. In such systems, the effect of multiple scattering of photons is quite important and has been identified to be in the root of long-range interactions, which allows for the system to be treated as a one-component plasma. Additionally, if the system is optically dense, the dynamics of the multiple scattered light can be diffusive, which leads to interesting optical features. In this work, by investigating the elementary excitations of the system, we show that the combination of these two effects can result on a mode spectrum containing a "roton" minimum.

In what follows, we consider that the dynamics of cold atoms in MOTs is described by the Vlasov equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla} + \frac{1}{m}\sum_i \mathbf{F}_i\cdot\boldsymbol{\nabla}_{\mathbf{v}}\right) f(\mathbf{r},v,t) = 0, \tag{1}$$

where $f(\mathbf{r},v,t)$ is the normalized distribution function. The total force $\sum_i \mathbf{F}_i = \mathbf{F}_T + \mathbf{F}_c$ accounts for both the trapping and cooling forces. The collective force can be described by a Poisson equation [1, 2], such that $\boldsymbol{\nabla}\cdot\mathbf{F}_c(\mathbf{r},t) = Q_{\text{eff}}\int d\mathbf{v} f(\mathbf{r},v,t)$. Here, we define the effective charge $Q_{\text{eff}} = \sigma_L(\sigma_R - \sigma_L)I/c$ of the atoms induced by light, where σ_R and σ_L represent the scattering and absorption cross sections [1, 3, 4, 5], and I is the light intensity. For most of the experimental conditions, the scattering cross section is larger than the absorption cross section, i.e. $\sigma_R > \sigma_L$, enforcing the effective charge to be a positive quantity. We have showed that the positiveness of Q_{eff} is an essential condition for the existence of stable oscillations in the system (see e.g. Ref. [2]).

The diffusive behavior of light inside the trap can be macroscopically described by the diffusion equation

$$\frac{\partial I}{\partial t} - \boldsymbol{\nabla}\cdot\mathcal{D}\boldsymbol{\nabla} I = 0. \tag{2}$$

The diffusion coefficient is determined by $\mathcal{D} = \ell^2/\tau$, where the photon mean free pass is $\ell = 1/n\sigma_L$, with $n = n_0\int f d\mathbf{v}$ standing for the atomic density. According to experimental results [6], the diffusion time τ can be considered as independent from the atom density, so the diffusion coefficient explicitly reads

$$\mathcal{D}(\mathbf{r},t) = \frac{1}{\sigma_L^2\tau^2 n^2} = \frac{1}{\sigma_L^2\tau^2 n_0^2}\left[\int f(\mathbf{r},\mathbf{v},t)d\mathbf{v}\right]^{-2}. \tag{3}$$

Linearization of the above equations, assuming periodic perturbations on both space and time, yields the following kinetic dispersion relation

$$1 = \frac{\omega_p^2}{k^2}\left(1 + \frac{\omega_d}{i\Omega - \mathcal{D}_0 k^2}\right)\int \frac{1}{v_z - \Omega/k}\frac{\partial f_0}{\partial v_x}\, d\mathbf{v}, \tag{4}$$

where $\mathbf{k} = k e_{\mathbf{z}}$. Here, we have defined two typical frequencies of the system. The first one is associated with the oscillations of the atoms due to the long-range force, corresponding to an effective plasma frequency [2], $\omega_p = \sqrt{\frac{Q_{\text{eff},0} n_0}{m}}$, where $Q_{\text{eff},0} = \sigma_L(\sigma_R - \sigma_L)I_0/c$. The second important quantity is the

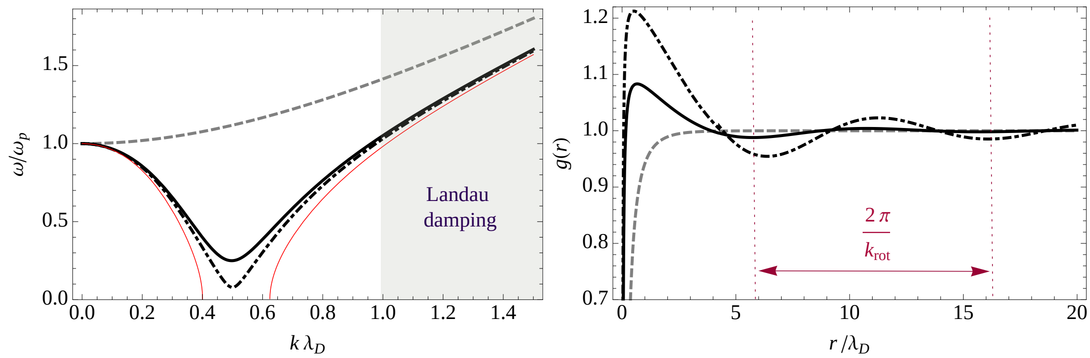

Fig. 1: Dispersion relation (left panel) and correlation function (right panel) depicted for $\mathcal{D}_0 = 2.0\lambda_D^2\omega_p$. $\omega_d = 0$ (gray dashed line), $\omega_d = 1.9\omega_p$ (black full line) and $\omega_d = 1.99\omega_p$ (black dashed line). A roton minimum emerges in $\omega(k)$ around $k_{\rm rot} = \lambda_D/\ell_d^2 = 0.5\lambda_D^{-1}$, being associated with the period $2\pi/k_{\rm rot}$ at which correlation function oscillates.

rate at which the photons scatter inside the trap, or simply the *diffusion frequency*, $\omega_d = \frac{2\nabla^2 I_0}{I_0}\mathcal{D}_0$. The integral in Eq. (4) can be evaluated using the Landau prescription, and assuming the atomic equilibrium to be described by a Maxwell distribution, $f_0(v) = \frac{1}{(2\pi v_{\rm th})^{3/2}} e^{-v^2/2v_{\rm th}^2}$, with $v_{\rm th} = \sqrt{k_B T/m}$ standing for the thermal speed, we obtain $1 = \frac{\omega_p^2}{\Omega^2}\left[\left(1+\frac{u_s^2 k^2}{\Omega^2}\right)\left(1+\frac{\omega_d}{i\Omega - \mathcal{D}_0 k^2}\right)\right] + i\pi\frac{\omega_p^2\Omega^2}{k^2}\left.\frac{\partial f_0}{\partial v_z}\right|_{v_z=\Omega/k}$, where we have defined the sound atomic speed $u_s = \sqrt{3}v_{\rm th}$. Separating the frequency into its real and imaginary parts, $\Omega = \omega + i\gamma$, with $\gamma \ll \omega$, we may finally write

$$\omega^2 = \left(\omega_p^2 + u_s^2 k^2\right)\left(1 - \frac{\omega_d \mathcal{D}_0 k^2}{\omega_p^2 + \mathcal{D}_0^2 k^4}\right) \quad \text{and} \quad \gamma = \frac{\omega_d}{2}\frac{\omega_p^2 + u_s^2 k^2}{\omega_p^2 + \mathcal{D}_0^2 k^4} - \frac{3}{\sqrt{8\pi}}\frac{1}{k^3\lambda_D^3} e^{-3/(2k^2\lambda_D^2)}, \tag{5}$$

where $\lambda_D = u_S/\omega_p$ is the effective Debye length. This dispersion relation describes a quasi-particle excitation resulting from the atom-photon coupling, or a polariton. An important property of the polariton mode described here is that they carry useful information about the long-range correlation of the system. From the dissipation-fluctuation theorem [7], the *dynamic* structure factor is given by $S(k,\omega) = (k_B T k^2/\pi\omega)\text{Im}\ \varepsilon(k,\omega)^{-1}$, where $\varepsilon(k,\omega) \equiv 1 + \chi(k,\omega)$ and $\chi(k,\omega)$ is the susceptibility. In the absence of hydrodynamic damping, the *static* structure factor $S(k) = \pi^{-1}S(k,\omega)\text{Re}\ [\varepsilon(k,\omega)^{-1} i\Omega]$ is finally given $S(k) = \frac{v_{\rm th}^2 k^2}{\omega(k)^2}$ [8]. The static two-point correlation function $g(r) = \langle n(r)n(0)\rangle/\langle n(r)\rangle\langle n(0)\rangle = \langle n(r)n(r')\rangle/n_0^2$ can then be easily calculated provided the relation $g(r) = 1 + \mathcal{F}^{-1}[S(k) - 1]$ [9], which after the integrating out the angular variables simply reads

$$g(r) = 1 + \frac{1}{\pi^2}\int_0^\infty \frac{k\sin(kr)}{r}\left[S(k) - 1\right]. \tag{6}$$

It is observed that the appearance of a minimum in the excitation spectrum (5) is associated with the occurrence of long-range correlation in the system (see Fig. (1)).

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