The Bohm criterion for flowing plasmas

D. M. Thomas[1], C. T. N. Willis[1], J. E. Allen (*)[1,2], M. Coppins[1]

[1] *Blackett Laboratory, Imperial College, London SW7 2BW, United Kingdom*
[2] *University College, Oxford OX1 4BH, United Kingdom*
(*) john.allen@eng.ox.ac.uk

The generalized Bohm criterion is discussed with relation to flowing plasmas, with particular reference to supersonic flow towards a large obstacle. Applications include flow onto a probe or a large dust particle. It is found that a small potential difference is developed in the plasma in order for sheath formation to take place.

Recent work on the supersonic flow of plasma onto a dust particle [1] has illustrated the fundamental difference between flow around an obstacle and flow which is absorbed on the surface. This makes apparent the necessity of further study of the plasma-sheath boundary. The well-known Bohm criterion for sheath formation [2] was derived for monoenergetic ions entering the sheath. Allowing for a distribution of ion energies Harrison and Thompson [3] derived the generalized Bohm criterion:

$$\int_0^\infty \frac{f(v)\,\mathrm{d}v}{v^2} \leq \frac{M}{kT_e} \tag{1}$$

In the present preliminary study we shall study a one-dimensional (plane) case and assume an initial drifting Maxwellian distribution of ion velocities

$$f(v) = \left(\frac{M}{2\pi kT_i}\right)^{1/2} \exp\left(-\frac{M(v-w)^2}{2kT_i}\right) \tag{2}$$

This distribution does not satisfy the above generalized Bohm criterion. Normalizing the velocities in terms of the ion thermal velocity $(kT_i/M)^{1/2}$, this distribution takes the form

$$f_1(\hat{v}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\hat{v}-\hat{w})^2}{2}\right) \tag{3}$$

If we accelerate the ions through a potential difference V, the distribution becomes

$$f_2(\hat{v}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\sqrt{\hat{v}^2-2\Phi}-\hat{w})^2}{2}\right) \tag{4}$$

where $\Phi = -eV/(kT_i)$. This follows from Liouville's theorem (or the Vlasov equation), which states that the value of $f(v)$ is constant along a particle trajectory. Figure 1 shows examples of the new velocity distributions. The integral of $f(v)$ is the ion density with respect to the original density, so we define a new distribution function $f(\hat{v}) = \alpha f_2(\hat{v})$ such that the integral is unity.

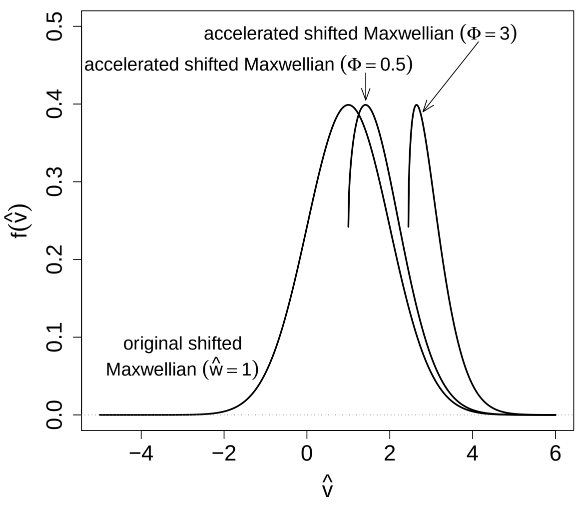

Fig. 1: Example velocity distribution functions with $\hat{w} = 1$, after accleration through potential differences of $\Phi = 0.5$ and $\Phi = 3$.

The generalized Bohm criterion can then be written in terms of the dimensionless distribution function

$$\int_0^\infty \frac{f(\hat{v})\,\mathrm{d}\hat{v}}{\hat{v}^2} \leq \beta \qquad \text{where } \beta = \frac{T_i}{T_e} \tag{5}$$

Taking the marginal (equality) form of this criterion we have

$$\int_{\sqrt{2\Phi}}^{\infty} \frac{\alpha}{\sqrt{2\pi}} \cdot \frac{1}{\hat{v}^2} \exp\left(-\frac{(\sqrt{\hat{v}^2 - 2\Phi} - \hat{w})^2}{2}\right) d\hat{v} = \beta \tag{6}$$

where

$$\int_{\sqrt{2\Phi}}^{\infty} \frac{\alpha}{\sqrt{2\pi}} \exp\left(-\frac{(\sqrt{\hat{v}^2 - 2\Phi} - \hat{w})^2}{2}\right) d\hat{v} = 1 \tag{7}$$

Equations 6 and 7 can be solved numerically to yield values of Φ; the resulting values of $|eV/(kT_e)|$ are shown in Figure 2. The average value of the new velocity distribution can then readily be found (see Figure 2, right, in terms of the original Bohm or ion acoustic velocity $c_s = (kT_e/M)^{1/2}$).

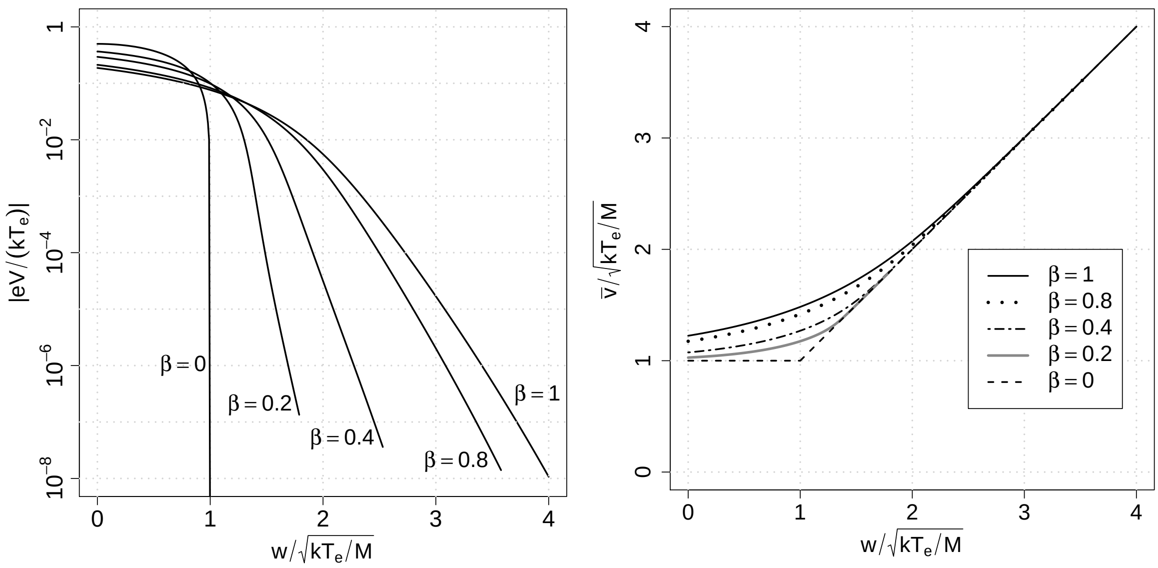

Fig. 2: The normalized potential difference $|eV/(kT_e)|$ required to satisfy the generalized Bohm criterion – for large drift velocities it is small (*left*); the average ion velocity on entering the sheath – it is little changed at high drift velocity (*right*).

Some discussion of the limitations of the present work should be given. The one dimensional (plane) geometry represents only an approximation to a real system. Flow around and onto an object of finite size should be considered. Another point is that quasi-neutrality exists at the plasma-sheath boundary in conventional theory of the Bohm criterion but there is a difference in electron and ion densities after acceleration through the potential difference V. This difference, however, is very small for appreciable flow velocities (Figure 2, left), so that the present calculations indicate that, at least for these velocities, the Bohm criterion can easily be satisfied.

References

[1] C. T. N. Willis, J. E. Allen, M. Coppins, M. Bacharis, *Phys. Rev. E* **84** (2011) 046410.

[2] D. Bohm, *The Characteristics of Electrical Discharges in Magnetic Fields* (1949), eds. A. Guthrie and R. K. Wakerling (New York: McGraw-Hill), chapter 3.

[3] E. R. Harrison, W. B. Thompson, *Proc. Phys. Soc.* **7** (1959) 145–152.