Two Dimensional Plasma Configurations

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We extend and correct recent communications concerned with two dimensional plasma configurations indicating how in cylindrical geometry magnetic fields affect commonly made assumptions and show how, in particular, the plasma balance equation, or as it is commonly called the global equation, can be generalized to cover a full range of pressures.

Introduction There is a growing interest in describing plasmas even magnetised ones in two dimensional configurations. An early example of such a situation is given in the worked examples in Lieberman and Lichtenberg [1] who gave the case of an ummagnetised plasma at higher pressures in a closed cylindrical vessel. Their plasma was in the Schottky or mobility-diffusion regime, for which in the plasma approximation the appropriate wall condition is zero density. Then one has to solve the equation $D_a\nabla^2 n + Z_i n = 0$, where D_a is the ambipolar diffusion coefficient, Z_i is is the ionization rate and n(r,z) is the plasma density. If the container has dimensions, radius = r_w, and length $2z_w$, then the solution is - $n = n_0 \cos(\pi z/2z_w)J_0(2.408r/r_w)$, n_0 being the central density, and the plasma balance equation is $-Z_i\nu_i/c_s^2 = \pi^2/4z_w^2 + 2.408^2/r_w^2$. c_s is the Bohm speed given by $c_s^2 = kT_e/M$.
Now a recent publication by Curelli and Chen [2] sought to describe a plasma in cylindrical geometry in which the plasma radial motion (particularly of the electrons) was significantly influenced by a uniform magnetic field B_z. This raises the question as to the extent we can develop earlier work to cover the full range of pressures and vessel dimensions used in plasma processing including a magnetic field.

At the same time Sternberg and Godyak [3] have given a treatment of an unmagnetised cylindrical plasma covering an intermediate range of pressures,essentialy extending the Lieberman and Lichtenberg problem.
This paper aims to push at the boundaries, is restricted to electropositive gases such as argon, and will include a magnetic field, but still is in the plasma approximation.

The Basic Equations One could write these out explictly and this has been done elsewhere (e.g. Franklin [4]) for positive-column-like geometry but the resulting equations are dauntingly complex and not obviously capable of solution in two dimensions even by the most sophisticated computational methods because - (i) at lower pressures the values of the plasma density at (near) the walls is finite, (ii) the particle motion of both the electrons and ions has an azimuthal (θ) component, (iii) it is now well established that the Boltzmann Relation which holds when it is justified in taking the limit the electron mass m→zero, is not valid when there is a magnetic field present because this would imply that the electron cyclotron frequency $\omega_{ce} = eB/m \rightarrow \infty$.

Our present approach This is to progressively relax the assumptions made so that one generalises the plasma balance (global) equation testing its limits at each stage.

Higher pressures applied magnetic field – In this case we draw on known results where the ambipolar diffusion coefficient for drift across the magnetic field lines is reduced by a factor $1/(1 + \omega_{ce}^2/\nu_e^2)$ (Bickerton and von Engel [5], ν_e is the electron collision frequency for momentum transfer, and so in cylindrical geometry our first equation is replaced by - $D_a\partial^2 n/\partial z^2 + D_{am}(\partial^2 n/\partial r^2 + \partial n/r\partial r) + Z_i n = 0$. This results in a plasma balance equation $-Z_i\nu_i/c_s^2 = \pi^2/4z_w^2 + 2.408^2 D_{am}/D_a r_w^2$, where $D_{am} = D_a/(1+\omega_{ce}/\nu_e^2)$.

Low pressures The basic equations in the absence of a magnetic field are of the form
$Z_i/c_s \sim 1/r_w$ or $1/z_w$ in cylindrical and in plane geometry and thus the heurestic analogues would be of the form $-Z_i^2/c_s^2 = \lambda^2_{lpp}/z_w^2 + \lambda^2_{lpc}/r_w^2$ where λ_{lpp} is the low pressure plane eigenvalue and λ_{lpc} is the low pressure cylindrical one, and both are available from Franklin [4].

Transitional pressures – Here we are in the midst of a multi-parameter regime and no simple expressions are likely to suffice but reasonable values can be interpolated from the limiting regimes, and there are values or expressions which allow such a process to be carried out. Thus we believe that we have captured over the whole parameter regime expressions which can allow reasonable estimates to be made of the important parameters determining processing plasmas.

It is possible to construct approximate equi-potentials and particle drifts on the vessel cross-section and even indicate how these vary with plasma collisionality and magnetic field strength, and thus attempt a relaxation method solution of such situations given boundary conditions and reasonable starting values, but we leave that to others.

Conclusions We cast doubt on much that is contained in Curelli and Chen because of their conclusion that the Boltzmann Relation applies in their plasma and furthermore it is not contained within a cylindrical space, cyclotron plasmas are essentially open-ended.
But we hope we will enable others to take the matters raised forward.

References

[1] M. A. Lieberman and A. J. Lichtenberg *Principles of Plasma Discharges and Plasma Processing* (2005) Wiley : Interscience page 382
[2] D.Curelli and F. F.Chen *Physics of Plasmas* **18**, (2011) 113501
[3] N. Sternberg and V. Godyak Plasma Sources Science and Technology 20, (2011)
[4] R. N. Franklin R N *J. Plasma Phys*. **77** (2011)
[5] R. J. Bickerton and A. von Engel *Proc. Phys. Soc. B* **69**, (1956) 468
[6] R. N. Franklin *Phys Plasmas* (2012) (in press)