On the description of electron transport in fluid models

M.M. Becker (*), F. Sigeneger, D. Loffhagen

Leibniz Institute for Plasma Science and Technology e.V., 17489 Greifswald, Germany
(*) markus.becker@inp-greifswald.de

A new drift-diffusion approximation for the electrons in nonthermal gas discharge plasmas is proposed. The transport coefficients for the electron energy involved in the new as well as in common approaches are compared with those provided by the so-called heat flux ansatz. The influence of the different approaches for the electron component is discussed using the example of a low-pressure glow discharge in argon.

Fluid models are commonly used for the theoretical description of low temperature plasmas including RF discharges, glow discharges as well as dielectric barrier discharges at low and atmospheric pressure. Nonlocal phenomena are inherent in all these kinds of discharge plasmas [1, 2]. In particular, in the cathode region and near local disturbances the nonlocal energy transport plays an important role.

Usually hybrid models, which treat kinetically only the electron component, have been applied for the description of nonlocal phenomena, since the electrons are mainly responsible for the energy transport within the plasma and common fluid models are not able to describe kinetic effects [2]. However, Robson *et al.* [3, 4] have pointed out that fluid models are able to describe nonlocal phenomena if the electron heat flux is properly defined. Considering only the axial dependence x, the ansatz proposed by Robson *et al.* for the electron heat flux $\dot{q}_\mathrm{e}$ takes the form [3]

$$\dot{q}_\mathrm{e}^\mathrm{HFA} = -\frac{2}{3}\partial_x\Big(\frac{\xi n_\mathrm{e}}{m_\mathrm{e}\nu_\mathrm{m}}\Big) - \frac{1}{3}(5-2\beta)\frac{\varepsilon e_0 n_\mathrm{e}}{m_\mathrm{e}\nu_\mathrm{m}}E - \frac{5}{3}\varepsilon\Gamma_\mathrm{e}\,. \tag{1}$$

Here, e_0, m_e, n_e, ε and Γ_e denote the elementary charge and the mass, particle density, mean energy and particle flux of the electrons, respectively, ν_m is the momentum-transfer collision frequency and E is the electric field. If only elastic collisions take place and $\nu_\mathrm{m} \sim \varepsilon$ the expression (1) is stated to be exact with $\xi = \varepsilon^2$ and $\beta = 1$. In more general cases, the parameter functions ξ and β have to be adapted carefully [3].

In the present contribution, expressions for the heat flux resulting from common drift-diffusion models have been examined. Starting from the electron kinetic equation, the consistent drift-diffusion approximation for the particle and energy flux can be derived by expanding the velocity distribution function of the electrons in Legendre polynomials. Supposing a quasi-stationary evolution of the first component f_1 to the distribution anisotropy, the expression [5]

$$\dot{q}_\mathrm{e}^\mathrm{DDAc} = -\partial_x\big(D_\varepsilon n_\mathrm{e}\big) - b_\varepsilon E n_\mathrm{e} - \frac{5}{3}\varepsilon\Gamma_\mathrm{e} \tag{2}$$

for the heat flux of the electrons finally results with the diffusion coefficient D_ε and mobility b_ε for the electron energy transport. For some reason, the consistent coefficients D_ε and b_ε are frequently simplified by [5]

$$D_\varepsilon = \frac{5}{3}\varepsilon D_\mathrm{e} \quad \text{and} \quad b_\varepsilon = \frac{5}{3}\varepsilon b_\mathrm{e}\,, \tag{3}$$

where D_e and b_e are the diffusion coefficient and mobility for the particle transport. Using the example of argon plasmas in a discharge arrangement with plane electrodes, it was found that the consistent approach (DDAc) using (2) is applicable to a limited range of discharge parameters only.

To overcome this restriction, a new, consistent drift-diffusion approximation (DDAn) has been derived. Starting from the macroscopic balance equations for the first four moments, namely particle density, particle flux, energy density and energy flux, and the expansion of electron velocity distribution function in Legendre polynomials, finally the expression

$$\dot{q}_\mathrm{e}^\mathrm{DDAn} = -\frac{1}{\tilde{\nu}_\mathrm{e}}\partial_x\Big(\tilde{\xi}_1 n_\mathrm{e}\Big) - \frac{e_0}{\tilde{\nu}_\mathrm{e}}\Big(\frac{5}{3}\frac{\varepsilon}{m_\mathrm{e}} + \xi_1\Big)E n_\mathrm{e} - \frac{5}{3}\varepsilon\Gamma_\mathrm{e} \tag{4}$$

for the electron heat flux is obtained, when assuming quasi-stationarity of the expansion coefficient f_1. The electron particle and energy transport coefficients in (2), (3) and (4) have been obtained by solving the steady state, homogeneous electron kinetic equation according to [6].

The impact of the different approaches for treating the heat flux of the electrons is illustrated in figure 1a for the mean electron energy in an abnormal argon dc glow discharge with plane-parallel electrodes separated by a gap of 1 cm. The steady state results have been obtained by means of a time-dependent fluid model similar to that represented in [6] using the local mean energy approximation. The pressure was 1 Torr and external voltages of $-250\,\mathrm{V}$ (top) and $-500\,\mathrm{V}$ (bottom) were applied at $x = 0$, while the anode at $x = 1\,\mathrm{cm}$ was grounded. The results of the three approaches differ substantially in the entire discharge region. In particular, the approaches DDAn and DDAc predict a minimum in the transition region from the cathode fall to the negative glow, which is caused by energy transport effects. Such minimum is not present in the results of the simplified approach DDA5/3 using (3). For the high voltage case the energy minimum in DDAc becomes negative and no steady state solution was obtained.

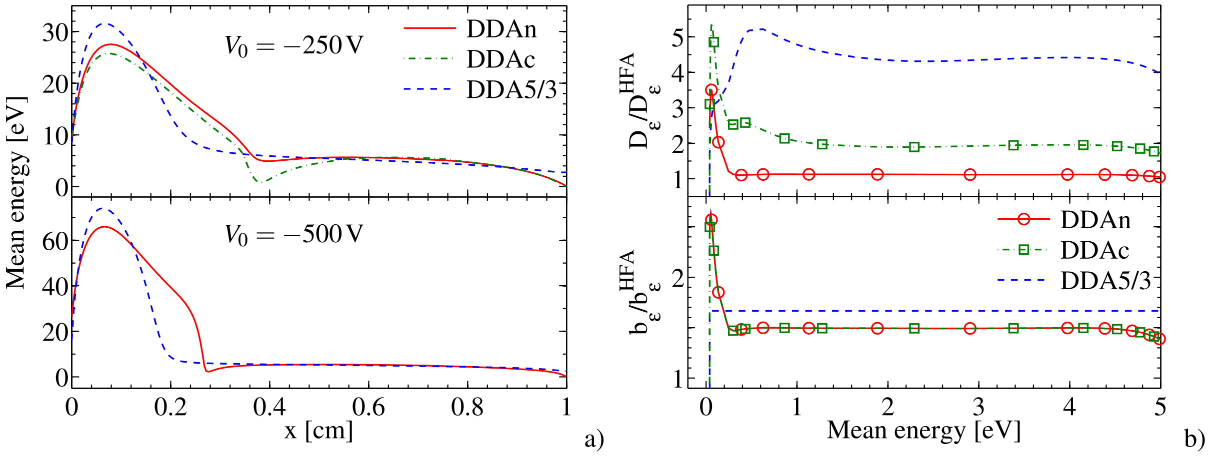

Fig. 1: Mean energy in an abnormal glow discharges in low pressure argon for different voltages V_0 (a) and comparison of energy transport coefficients as functions of the mean energy (b).

Figure 1b shows the ratios $D_\varepsilon/D_\varepsilon{}^{\mathrm{HFA}}$ (top) and $b_\varepsilon/b_\varepsilon{}^{\mathrm{HFA}}$ (bottom) of the energy diffusion coefficient D_ε and mobility b_ε to the coefficients $D_\varepsilon{}^{\mathrm{HFA}} = 2\varepsilon^2/(3m_\mathrm{e}\nu_\mathrm{m})$ and $b_\varepsilon{}^{\mathrm{HFA}} = e_0\varepsilon/(m_\mathrm{e}\nu_\mathrm{m})$ of the heat flux ansatz (1) for the coefficients in (2), (3) and (4), named $D_\varepsilon{}^{\mathrm{DDAc}}$, $b_\varepsilon{}^{\mathrm{DDAc}}$, $D_\varepsilon{}^{\mathrm{DDA5/3}}$, $b_\varepsilon{}^{\mathrm{DDA5/3}}$, $D_\varepsilon{}^{\mathrm{DDAn}} = \tilde{\xi}_1/\tilde{\nu}_\mathrm{e}$ and $b_\varepsilon{}^{\mathrm{DDAn}} = e_0\left(5\varepsilon/(3m_\mathrm{e}) + \xi_1\right)/\tilde{\nu}_\mathrm{e}$, respectively. It points out that the coefficient $D_\varepsilon{}^{\mathrm{DDAn}}$ of the new drift-diffusion approximation DDAn deviates by less than 20% from $D_\varepsilon{}^{\mathrm{HFA}}$ at mean electron energies between 0.3 and 5 eV, where mainly elastic collisions take place, while the consistent coefficient $D_\varepsilon{}^{\mathrm{DDAc}}$ is about 100% larger than $D_\varepsilon{}^{\mathrm{HFA}}$ for $\varepsilon \geq 1\,\mathrm{eV}$. Even larger deviations are found for $D_\varepsilon{}^{\mathrm{DDA5/3}}$. The energy mobility b_ε of the drift-diffusion approximations differ about 50% from the coefficient $b_\varepsilon^{\mathrm{HFA}}$. This points out that the assumptions inherent in the models have to be questioned and investigated in more detail.

Acknowledgments

The work was supported by the Deutsche Forschungsgemeinschaft with SFB TRR 24.

References

[1] L. D. Tsendin, *Phys.-Usp.* **53** (2010) 133-157.
[2] H. C. Kim *et al.*, *J. Phys. D: Appl. Phys.* **38** (2005) R283-R301.
[3] R. E. Robson, R. D. White, Z. Lj. Petrović, *Rev. Mod. Phys.* **77** (2005) 1303-1320.
[4] P. Nicoletopoulos, R. E. Robson, *Phys. Rev. Lett.* **100** (2008) 124502.
[5] G. J. M. Hagelaar, L. C. Pitchford, *Plasma Sources Sci. Technol.* **14** (2005) 722-733.
[6] G. K. Grubert, M. M. Becker, D. Loffhagen, *Phys. Rev. E* **80** (2009) 036405.