Shear driven instabilities in dusty plasmas

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Most of the space plasma is dusty. It contains charged grains and, the coupling of the grains to the magnetic field determines the nature of wave propagation in the medium. The plasma-dust collision can causes not only damping of the high frequency waves but also assist the excitation and propagation of low frequency fluctuations in the medium. Therefore, it should be anticipated that the plasma–dust collision, in the presence of shear flows may open new channels through which free shear energy (if it is available in the plasma) can flow to waves. The large scale structures in space such as clumps and filaments often provide motivation to investigate the behaviour of a dusty medium. Since the propagation of low frequency fluctuations in a collisional medium can become unstable in the presence of shear flow, it is likely that such waves, when unstable, can provide the right conditions for formation of clumps. We carry out comparative study of known hydrodynamic and magnetohydrodynamic shear driven instabilities in dusty plasma by comparing their maximum growth rate and the parameter window in which they operate.

The collisional process strongly influences diffusion of the magnetic field and wave propagation in a dusty plasma. Therefore, it should be anticipated that the plasma–dust collision which *unfreezes* the magnetic flux from the fluid, may open up new channels through which free shear energy (if it is available in the plasma) can flow to the waves. Therefore, it is desirable to investigate the shear instability of magnetised collisional dusty plasma. The Kelvin-Helmholtz instability (KHI) of magnetized three–component plasma consisting of electrons, ions and charged dust was investigated in the past [2]. However, since analysis in Ref. [2] assumes that plasma particles are Boltzmannian, it neglects the transverse (to the field) drift of the plasma particles against the dust. This relative drift between the magnetised plasma and the dust is at the heart of Hall electric field in dusty plasmas [1]. Due to the large difference between the mass of the dust ($\sim 10^{-15} - 10^{-5}$ g) and plasma ($m_e \sim 10^{-27}$ g, $m_i \sim 10^{-24}$ g) particles, there is physical ground to expect the presence of relative transverse motion between the plasma particles and charged dust in the magnetized dusty medium. The relative drift between lighter, magnetized plasma components against charged dust causes local charge separation which in turn will give rise to the generation of transverse electric field. Therefore, Hall electric field will always be present in a dusty plasma [1]. We shall show that the non-thermal character of the plasma particles which manifests itself as Hall diffusion of the magnetic field couples with the shear flow and drives new instability in the medium. It is important to distinguish it from well know shear driven Kelvin-Helmholtz instability. Unlike KHI which is hydrodynamic in nature, this new, Hall effect driven shear instability is magnetohydrodynamic and expectedly disappears in the absence of magnetic field.

The multi-component dusty plasma consists of electrons, ions and charged dust. The basic set of equations for a three component, collisional, magnetized dusty fluid has been described in the past [3] and thus, we shall only briefly describe them here. We shall define mass density of the bulk fluid as $\rho = \rho_e + \rho_i + \rho_d \approx \rho_d$. Then the bulk velocity $\mathbf{v} = (\rho_i \mathbf{v}_i + \rho_e \mathbf{v}_e + \rho_d \mathbf{v}_d)/\rho \approx \mathbf{v}_d$. Thus,the Continuity, momentum and induction equation becomes

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\,\mathbf{v}) = 0\,. \quad \rho\,\frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J}\times\mathbf{B}}{c}\,, \frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left[(\mathbf{v}_d\times\mathbf{B}) - \eta_H\left((\nabla\times\mathbf{B})\times\hat{\mathbf{B}}\right)\right]. \qquad (1)$$

Here $P = P_e + P_i + P_d$ is the total pressure, current density $\mathbf{J} = e\,(n_i\,\mathbf{v}_i - n_e\,\mathbf{v}_e + Z\,n_d\,\mathbf{v}_d)$, $\hat{\mathbf{B}} = \mathbf{B}/B$ and $\eta_H = cB/4\pi Z e n_d$ is the Hall diffusion. The Hall diffusion, η_H is proportional to the magnetic field and is inversely proportional to the dust charge. When the ion kinetic energy is negligible in comparison with the ion thermal energy the charge on dust can be approximated as [4]

$$Z\,e \approx \frac{4}{\pi}\,\frac{a\,k_B\,T}{e}\,,\text{which gives } \eta_H = 6.5\times 10^{15}\,\frac{B}{a\,n_d\,T}\,\mathrm{cm}^2\,\mathrm{s}^{-1}\,. \tag{2}$$

Thus implies that Hall diffusion increase with grain size as $\eta_H \propto a$. In interstellar medium, extinction curve suggest that dust number density scales with dust radius as $n_d(a) \propto a^{-3.5}$. In this case, $\eta_H \propto a^{2.5}$. This is not surprising since larger is the dust, stronger is the relative drift between the plasma and dust. This will result in stronger Hall electric field. Thus we see that whereas Hall diffusion increases linearly with magnetic field, its dependence of dust radius is quadratic. As a result, even for an extremely weak magnetic field, strong Hall diffusion can be induced in cold dusty plasma experiments.

We shall assume an initial homogeneous state with an azimuthal shear flow $\mathbf{v}_d = v_0{}'\,x\,\mathbf{y}$ and an uniform vertical field, $\mathbf{B} = B\,\mathbf{z}$. To keep the analysis analytically transparent, we shall further assume that the fluid is incompressible. Linearising Eqs. (1) with perturbation of the form $\exp\left(\sigma t + i\,k\,z\right)$, yields following dispersion relation

$$\hat{\sigma}^4 + \hat{k}^2\left(2 - \alpha\,\hat{\eta}_H + \hat{k}^2\,\hat{\eta}_H^2\right)\hat{\sigma}^2 + \hat{k}^4\left(1 - \alpha\,\hat{\eta}_H\right) = 0\,. \tag{3}$$

In the above Eqn., wavenumber k, frequency σ and diffusivity η are normalised: $\hat{k} = k\,v_A/|v_0'|$, $\hat{\sigma} = \sigma/|v_0'|$ and $\hat{\eta}_H = \eta_H\,|v_0'|/v_A^2$. Here $v_A = B/\sqrt{4\pi\rho}$ is the Alfvén speed and $\alpha = -v_0'/\,|v_0'| \equiv \pm 1$. The necessary condition for the instability can be written as $1 - \alpha\,\hat{\eta}_H < 0$. The quantity $1 - \alpha\,\hat{\eta}_H$ is the magnetovorticity and is frozen in the fluid. Therefore, the necessary condition for the instability reduces to the requirement of negative magnetovorticity. Since $\hat{\eta}_H$ can change sign depending on the direction of the local vertical field with respect to the flow gradient, preceding necessary condition can be easily satisfied for given flow gradient.

The maximum growth rate of the instability becomes $\sigma_0 = 0.5\,|v_0'|$. Note that the maximum growth rate is proportional only to the shear gradient and is independent of the field strength. This is similar to KHI. Assuming $\alpha = 1$, in the dimensional form, most unstable wavenumber can be written as

$$k_0^2 = -\frac{1}{2}\left[\frac{n_d\,a\,T\,\left(v_0'\right)^2}{2\,n_d\,a\,T\,v_A^2 - 6.5\times 10^{15}\,v_0'\,B}\right]. \tag{4}$$

It is clear from Eq. (4) that when Hall diffusion is the dominant term in the above equation, the maximum wavelength is proportional to the dust radius and inversely proportional to the square root of plasma temperature. For $\lambda_0 \propto a$. Thus in low temperature dusty plasmas, long wavelength fluctuations will grow at maximum rate. Although the maximum growth rate of the present instability is similar to KHI, the most unstable wavelength differs with KHI. An approximate expression for most unstable wavelength for KHI is [5] $\lambda_0 \sim 8\,h$ where $h = |v_0|/|v_0'|$ is the shear scale.

To summarise, the generation of low frequency waves in the dusty plasma in the presence of shear flow can become unstable due to the relative drift between the plasma and dust particles. The sign of the shear gradient and the direction of local vertical magnetic field set the condition for the onset of the instability. In the present work we have assumed that dust dynamics is important. However, even when dust is immobile, the relative transverse drift between the plasma particles will cause Hall field, and thus the condition for the shear instability can be easily met. The relative drift between the different plasma component and thus ensuing Hall field is crucial to this instability.

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