Plasma magnetisation and Farley-Buneman Instability

B. P. Pandey[1(*)], S. V. Vladimirov[2]

[1] *Department of Physics & Astronomy, Macquarie University, NSW 2109, Australia*
[2] *School of Physics, The University of Sydney, NSW 2006, Australia*
[(*)] birendra.pandey@mq.edu.au

The Farley-Buneman instability (FBI) which is electrostatic streaming instability is thought to play important role in creating plasma irregularities in the ionospheric E-regions. The interplay between electric and magnetic field which give rise to cross field motion in the ionised medium causes this instability. In a weakly ionised medium, collision between neutral and plasma particles is important and relative magnetisation determines the magnitude of relative cross-field motion. Therefore, the role of plasma magnetisation on FBI should be investigated. The role of plasma magnetisation and neutral in exciting FBI in magnetised plasma is investigated.

The Farley–Buneman instability (FBI) has been extensively studied in theory for over last several decades. It is well-known that once the relative drift of ions and electrons exceeds the ion-acoustic speed, the interactions between waves and plasma will lead to the growth of the instability [1, 2]. It is well established that field–aligned plasma irregularities in the E region of the Earths equatorial ionosphere are associated with the electrojet current. The FBI is thought to create plasma irregularities in the ionospheric E-region where electrons are strongly magnetized. The interplay between Earths electric and geomagnetic field produces currents which give rise to this instability. When the electrons are strongly magnetized, the collisional drag of the ions by neutral flows can also cause the development of a similar instability

However, the studies of the FBI under general space conditions are incomplete as they do not take into account following important effects: (a) the presence of dust is often neglected. It is well known that dust not only affect the fractional ionisation but due to its weak magnetisation, strongly couples to the neutrals; (b) the space plasma is generally partially ionised with overwhelming presence of neutral particles. Clearly, the presence of charged dust owing to its large inertia can cause Hall effect in the plasma. Further, the presence of neutrals will excite low frequency fluctuations in the medium. Therefore, it is important to consider the dynamics of all the constituents of weakly ionised matter before investigating the stability of such a plasma. It is well know that when the finite magnetization of ions is taken into account, the Hall diffusion weaken the FBI, and the system becomes stable for any neutral flow velocity when the ion is strongly magnetized [[3].

In general, weakly ionised plasma can be modelled by following equations

$$D\vec{\rho}+\rho\,\vec{\nabla}\cdot\vec{v}=0\,,\quad D_j\,\vec{v}_j=-\frac{\vec{\nabla}P_j}{\rho_j}+\frac{q_j}{m_j}\left[\vec{E}+\frac{\vec{v}_j\times\vec{B}}{c}\right]-\nu_{jn}\left(\vec{v}_j-\vec{v}_n\right)\,,$$
$$D_n\,\vec{v}_n=-\frac{\vec{\nabla}P_n}{\rho_n}+\nu_{jn}\left(\vec{v}_j-\vec{v}_n\right)\,. \qquad (1)$$

where $D_j=\partial/\partial t+\vec{v}_j\cdot\vec{\nabla}$, $\vec{v}_j\,,rho_j$ is the plasma velocity and density, P_j is plasma pressure, $\vec{E}$ and $\vec{B}$ are the electric and magnetic field, c is the speed of light and j stands for electron, ion and dust with $q_e=-e\,,q_i=e\,,q_d=Z\,e$. The last equation describes the neutral dynamics. We shall close the above set of equation by assuming $P_j=c_j^2\,\rho$ where $c_j^2=k_B\,T_j/m_j$ is the thermal speed. The collision frequency in the above equation is $\nu_{jn}=\rho_n\gamma_j$ with

$$\gamma_j=\frac{<\sigma v>_j}{m_i+m_n}\,, \qquad (2)$$

and $<\sigma v>_j$ is the rate coefficient of collisional momentum exchange. We shall assume weakly ionized plasma and neglect plasma inertia in Eq. (1). Further, we shall assume cold, massive, micron size unmagnetized dust, i.e. $\beta_d\ll 1$. Here magnetization is defined by the plasma Hall parameter $\beta_j=\omega_{cj}/\nu_{jn}$

which is a ratio of the Lorentz and the collisional exchange terms in the momentum equation. Here $\omega_{cj} = q_j B/m_j c$ is the cyclotron frequency. We further assume that the plasma is threaded by a uniform magnetic field B in the z direction and that the neutrals have transverse background velocity $v_n = v_n \vec{y}$. Equations (1) then yield a stationary solution for the background plasma drift velocities

$$\vec{v}_{i\perp} = \frac{\vec{v}_{n\perp} + \beta_i \vec{v}_{n\perp} \times \vec{z}}{1+\beta_i^2}, \quad \vec{v}_{e\perp} = \frac{\vec{v}_{n\perp} - \beta_e \vec{v}_{n\perp} \times \vec{z}}{1+\beta_i^2}, \quad \vec{v}_{d\perp} = \frac{\vec{v}_{n\perp} + \beta_d \vec{v}_{n\perp} \times \vec{z}}{1+\beta_d^2}. \tag{3}$$

where $\vec{z}$ is the unit vector along z direction. Since $\beta_d \ll 1$, $\vec{v}_{d\perp} \approx \vec{v}_{n\perp}$. Therefore, the relative velocity between the ions, electrons and dust becomes $\vec{v} = \vec{v}_i - \vec{v}_e + \vec{v}_d$. On this background, we study linear electrostatic perturbations in the plane perpendicular to the background field. Linearising Eqs. (1) and Fortier transforming with $\vec{k} = (0,k,0)$ and neglecting terms $\sim m_e/m_d, m_i/m_d$ we get following dispersion relation

$$\left(\alpha_i \nu_{ne} \delta_e + \frac{m_e}{m_i} \alpha_e \nu_{en} \delta_i\right) \nu_{ni} \bar{\omega} + i C_1 = 0,$$

$$C_1 = \left(\nu_{ni} \nu_{in} \beta_i^2 + k^2 c_i^2\right) \nu_{ne} \delta_e + \frac{m_e}{m_i} \left(\nu_{ne} \nu_{en} \beta_e^2 + k^2 c_e^2\right) \nu_{in} \delta_i - \frac{m_e}{m_i} \beta_i \beta_e \nu_{en} \nu_{ne} \nu_{ni}, \tag{4}$$

$\bar{\omega} = \omega - k v, \alpha_i = 1 + \beta_i^2, \alpha_e = 1 + \beta_e^2$, $\delta_{e,i} = 1 \pm \frac{m_{e,i}}{m_d} \frac{Z \nu_{en,in}}{\nu_{dn}}$. Note that $\delta_e \approx 1$.

The above dispersion relation gives $\omega_r = k v$ and

$$\omega_i = \frac{(m_e/m_i)\, \delta_i \left[\beta_i \beta_e \nu_{en} \nu_{ne} \nu_{ni} - \nu_{in} \left(\nu_{ne} \nu_{en} \beta_e^2 + k^2 c_e^2\right)\right] - \nu_{ne} \delta_e \left(\nu_{ni} \nu_{in} \beta_i^2 + k^2 c_i^2\right)}{\left(\alpha_i \nu_{ne} \delta_e + \frac{m_e}{m_i} \alpha_e \nu_{en} \delta_i\right) \nu_{ni}}. \tag{5}$$

We note that $\delta_e \approx 1$. In general, above expression gives damping of the waves provided $\delta_i > 0$. However when $\delta_i \approx -\frac{Z m_i}{m_d} \frac{\nu_e n,in}{\nu_{dn}}$, for weakly ionised ions ($\beta_i \ll 1$ ω_i becomes

$$\omega_i \approx Z \frac{m_i n_n^2}{m_d n_i n_e} \beta_e \nu_{ne} + \frac{n_n}{n_i} \frac{k^2 c_s^2}{\nu_{ne}}. \tag{6}$$

Here $c_s^2 = k_B T_e/m_i$. While writing preceding equation we have used $\rho_n \nu_{ni} = \rho_i \nu_{in}$. From the above equation it is clear that the growth rate of the instability appears to be caused purely by collision and streaming seems to play no role. However, the streaming indirectly pumps the energy to electrostatic fluctuations through neutrals which drags immobile charged dust. Therefore streaming neutral directly couples to the field via dust. Therefore, it is not surprising that in the absence of dust, this instability disappears. In the mesosphere where charged dust plays important role in the formation of ice particles, above expression suggest the instability could be important in structure formation in this region. With increasing altitude $\gtrsim 100\,\mathrm{km}$, the presence of dust becomes less important in the Earths ionosphere. In the absence of dust and without neutral dynamics $\omega_r \approx 0$ and ω_i in $\beta_e \beta_i < 1$ limit becomes

$$\omega_r \approx \beta_e \beta_i k v, \quad \omega_i \approx \beta_e^2 \beta_i^2 \frac{(k v)^2}{\nu_{in}} - \frac{k^2 c_s^2}{\nu_{in}}, \tag{7}$$

which is modified FBI growth rate. Therefore, when the ions are weakly magnetised, the instability is quenched by the ion-neutral collision. When ions are unmagnetized, FBI completely disappears due to dissipative loss of free streaming energy by the ion-neutral collision.

To summarise, when neutral dynamics is properly considered, and plasma is highly collisional and dusty in nature, the growth rate of when ions are weakly magnetised is independent of streaming velocity. Streaming only indirectly pumps the energy to the fluctuations. In the absence of dust, when neutral dynamics is neglected, the growth rate is proportional to the plasma magnetisation as well as streaming velocity. When ion is unmagnetised, the instability disappears. Therefore, in the absence of dust, ion magnetisation is necessary to excite FBI like instability in the medium.

[1] O. Buneman, *Phys. Rev. Lett.* **10** (1963) 285–287.

[2] D. T. Farley, , *J. Geophys. Res.* **68** (1963) 6083–6097.

[3] B. G. Fejer, J. Providakes & D. T. Farley, , *J. Geophys. Res.* **89** (1984) 7487–7494.