Gaussian superposition model for electron density profiles of hypersonic entries in Earth's atmosphere

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Due to the difficulties in incorporating a full detailed kinetic model into the diffusion entry problem, we present the solutions obtained for the one-dimensional case using the Green's function method to solve the inhomogeneous diffusion equation subject to a particle source term accounting for ionization and recombination of electrons. The calculations are applied to the entry in Earth's atmosphere at $T = 10^4$ K and $p = 4.5\text{x}10^3$ Pa, assuming either free diffusion or ambipolar diffusion.

The aerothermodynamics analysis of the high-altitude hypersonic entry trajectories into the Earth's atmosphere are routinely performed using direct Monte Carlo simulation [1,2]. Such method constituted a very hard task for the computer resources of twenty years ago [1] but nowadays this is perfectly feasible, although special numerical schemes are still needed to generate a sufficient number of charged particles (ions and electrons) [2]. This difficulty arises from the much smaller mass of electrons when compared with the heavy air species. However, these models as well as the fluid models are necessarily limited by the detail of the chemical kinetics associated being totally impracticable if a full state-to-state reaction model is considered. In fact, the appropriate simulations of vibrational exchange and dissociation processes in high-temperature plasmas (in the range 10^3-10^5 K) mandates the application of detailed state-to-state models being the total number of reactions to be considered very large [3]. Moreover, an accurate analysis of the flow profiles along the stagnation streamline, under the combined effects of ionization, recombination and diffusion, should consider the transition from free to ambipolar diffusion as the electron density increases [4]. With these facts in mind and with the aim of incorporating a detailed kinetics into the diffusion models we present in this paper the results obtained for the reentry problem as a superposition of appropriate Gaussian solutions for the diffusion equation linked to the volume kinetics is considered.

The solution to the one-dimensional homogeneous diffusion equation, i.e. without a forcing term, in the case of a semi-infinite domain $0 \leq x < \infty$ initially populated to a unit density along a finite interval $[a, b]$, with $0 < a < b$, and assuming that at $x = 0$ the density is held at zero and vanishes at infinity can be obtained from a standard Gaussian solution using the method of images and superposition. This yields the solution:

$$g(x,t) = \frac{1}{2}\left\{ \operatorname{erf}\left(\frac{x+a}{2\sqrt{Dt}}\right) + \operatorname{erf}\left(\frac{x-a}{2\sqrt{Dt}}\right) - \operatorname{erf}\left(\frac{x+b}{2\sqrt{Dt}}\right) - \operatorname{erf}\left(\frac{x-b}{2\sqrt{Dt}}\right)\right\} \ .$$

where *erf(z)* is the error function and D is the diffusion coefficient. Then, the Green's function method can be used to obtain the solution to the inhomogeneous diffusion equation in the case of a particle source term *R(t)* including ionization and recombination of electrons, as follows:

$$\frac{\partial n_e}{\partial t} - D\,\frac{\partial^2 n_e}{\partial x^2} = R(t) \implies n_e(x,t) = \int_0^t R(t')\; g(x, t-t')\; dt' \ .$$

In the running conditions considered here ($T = 10^4$ K, $p = 4.5\text{x}10^3$ Pa) the dominant ionization channels are collisions N + N, O + O, and N + O, while the electron recombination takes place under the form of three-body processes to N^+ and O^+ ions [2,5]. This means that the ionization rate R_{ion} is independent of the electron

density. If further a two-order recombination process is assumed for simplification, with a rate $R_{rec}= \nu\, n_e$ and where $\nu = k\, n_{eav} = const.$ is the recombination frequency, with k denoting the three-body recombination rate coefficient and n_{eav} a somewhat average electron density, we get the simplified solution:

$$n_e(x,t) = R_{ion} \int_0^t g(x, t-t')\, e^{-\nu(t-t')}\, dt' \,.$$

The calculations are realized assuming that the ionization takes place originally within the interval [a, b], with a = 4 cm and b = 5 cm, which is the region where the concentrations of N and O atoms are most appreciable, and the vehicle surface is at x = 0 where n_e = 0 is held. For the purposes of this very single study, the concentrations of neutral and ionic species have been obtained from 2-D CFD simulations by considering the maxima of these concentrations along the stagnation streamline [6].

Fig.1 shows the space distribution of the ratio of the electron density to the gas number density, n_e/N, in the interval x = 0-15 cm, in the case of free diffusion of electrons, at the instants $t = 10^{-6}$, 10^{-5}, 10^{-4} and 10^{-3} s. Since the electron density is initially confined to 4–5 cm interval, it rapidly spreads out as the time evolves reaching its steady state value at ~10^{-4} s. The relative electron density continuously grows from a maximum of 3.5x10^{-4} at $t = 10^{-6}$ s to 1.7x10^{-3} at $t = 10^{-4}$ s. In turn, Fig.2 shows the same information for the case of ambipolar diffusion of electrons. We see that using the ambipolar diffusion coefficient there is a much smaller propagation of the plasma front, and a much sharper plasma front. Moreover, the central maxima are now more pronouced. These results are in line with those obtained for an ionization-diffusion plasma front propagation in a microwave field [7].

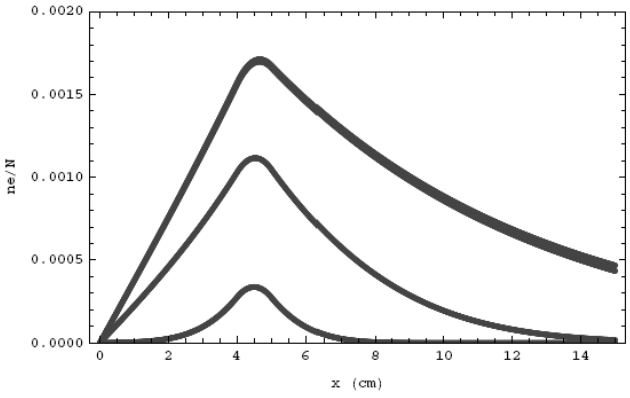

Fig.1: Ratio of the electron density to the gas number density in the range x = 0 (vehicle) – 15 cm, in the case of free electron diffusion, at the instants $t = 10^{-6}$, 10^{-5}, 10^{-4} and 10^{-3} s. The last two curves are nearly undistinguished.

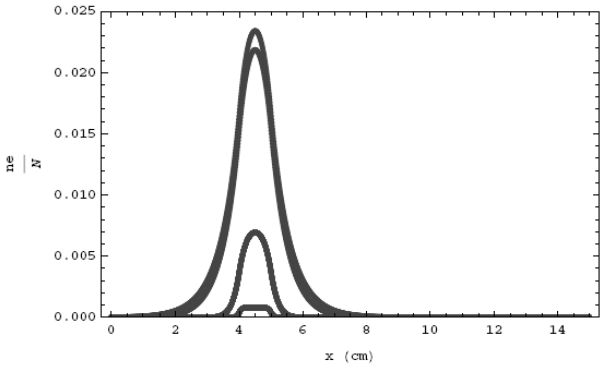

Fig.2: As in Fig.1 but for the case of ambipolar diffusion. The instants are the same as in Fig.2.

We can conclude that this method seems to constitute a very reasonable model to analyse the physics of electron diffusion in an entry plasma, something which is typically beyond what can be achieved with direct Monte Carlo simulation or CFD models. The method can still be improved by considering a smooth transition from free electron to ambipolar diffusion as the electron density increases.

References

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